The calibration pattern **140** has at least four or more point or line correspondences between the projector's output image and the camera's input image. The correspondences are used to determine two distinct homographies for the plane, one for each pose.

Projective Reconstruction

The two homographies are used to construct a scene up to a projective transformation as follows. Given the output calibration image of the projector **100** and the input image of the calibration pattern **140** acquired by the camera **110**, the correspondences in the output and input images are related by a 3×3 homography matrix H. If m_1 and m2 are projections of a 3D point M which belongs in the plane II, then

$$m_2 \sim= H\, m_1,$$

where m_1 and m_2 are homogeneous coordinates and ~= means equality up to scale.

Given two distinct planes (poses), and hence two distinct homographies, the epipoles e_1 and e_2 in the output and input images are determined using a generalized eigenvalue equation

$$(e_1 \neq =)\ kH^{-1}{}_1 e_2 = H^{-1}{}_2 e_2$$

where k is an unknown scalar.

For the projective reconstruction of the scene, the perspective projection matrices of the projector and the camera are then determined 240 as follows,

$$P_{1p}=[1\ 1\ 0]\ P_{2p=[H}\ 1\ e_2],$$

where H is one of the homographies. In our system, P_{1p} defines the projection matrix for the camera **110**. and P_{2p} the projection matrix for the projector **100**.

Euclidean Reconstruction

Next, the projection matrices are upgraded to give a Euclidean reconstruction. The goal is to find a 4×4 transformation 250 matrix G_p 260 such that

$$P_{1e}=P_{1p}G_p \sim= A_1[I1\ 0],\ \text{and}$$

$$P_{2e}=P_{2p}G_p \sim= A_2[R\ 1\text{-}Rt],$$

where A_1 is a 3×3 matrix describing the known camera intrinsic, A_2 is a 3×3 matrix describing unknown projector intrinsic, and rotation R and translation t define the relative physical relationship between the projector **100** and the camera **110**, up to unknown scale.

A_1 is known and can be factored out, so the goal is to find the matrix G 260 such that

$$P_{1e}=P_{1p}G \sim= [I1\ 0],\ \text{and} \tag{1}$$

$$P_{2e}=P_{2p}G \sim= A_2[R1\text{-}Rt].$$

The most general form of A_2 involves five intrinsic parameters: focal length, aspect ratio, principal point and skew angle, ignoring radial distortion. It is reasonable to assume that a projector has unity aspect ratio and zero skew.

Our method uses a simplified for of A_2 which involves two intrinsic parameters, the focal length R, and the vertical offset d of the principle point from the image center C. The assumption that the principal point is close to the center of the image as shown in FIG. **3***a*, common for prior art camera calibration techniques, is not true for projectors where the principal point usually has a substantial vertical offset d from the image center C, see FIG. **3***b*.

Thus, A_2has the form:

$$A_2=[f\ 0\ 0;0\ f\ d;0\ 0\ 1].$$

where f is the focal point, and d is the vertical offset of the principle point.

From equation (1), G is of the form

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ n1 & n2 & n3 & 1 \end{bmatrix}.$$

Here. the vector n=[n1 n2 n3]T defines the plane at infinity in the projective coordinate frame. The goal is to find n, and hence G. If G' denotes the first three columns of G, then it follows that $P_{2p}G' \sim= A_2$ R. Hence,

$$P_{2p}G'\,G'^T P_{2p}{}^T \sim= A_2RR^TA_2{}^T=A_2A_2{}^T$$

This leads to

$$P_{2P}\begin{bmatrix} I_{3\times3} & n \\ n^T & n^Tn \end{bmatrix} P_{2P}^T \cong A_2A_2^T, \tag{2}$$

where $K_2=A_2A_2{}^T=[f^20\ 0;0(f^2+d^2)d;0d\ 1]$.

Equation (2) is used to generate three constraints on the three unknowns of n. Two of the constraints, ($K_2(1,2)=0$ and $K_2(1,3)=0$), are linear in n1, n2, n3, and n^Tn. The third constraint ($K_2(2,\ 2)\ \text{-}K_2(1,\ 1)\ \text{-}K_2(2,3)_2 =0$) is quadratic. Hence, it is possible to express n1, n2, and n3 in terms of n^Tn.

Using quadratic constraint ($n1^2+n2^2+n3^2=n^Tn$) generates four solutions for the three unknowns of n. Each solution is used with equation (2) to determine 270 the intrinsic parameters f and d, and equation (1) is used for R and t.

Physically impossible solutions, e.g., solutions in which observed scene points are behind the camera, are eliminated to give a single solution for the true A_2, R and t.

This invention is described using specific terms and examples. It is to be understood that various other adaptations and modifications may be made within the spirit and scope of the invention. Therefore, it is the object of the appended claims to cover all such variations and modifications as come within the true spirit and scope of the invention.

What is claimed is:

1. A method for calibrating a projector with a camera being a fixed physical relationship relative to each other, comprising:

projecting an output image onto a display surface for a first and second pose of the projector and the camera relative to a display surface;

acquiring, for each pose, an input image;

determining, for each pose, a projector perspective projection matrix and a camera perspective projection matrix from each input image;

determining, for each pose, a transformation from the projector perspective projection matrix and the camera perspective projection matrix to Euclidean form; and

deriving the projector intrinsic parameters from the transformations.

* * * * *